

MATHEMATICAL MODELLING OF NON-LINEAR PROCESS USING SYSTEM IDENTIFICATION AND ROBUST CONTROLLER DESIGN

Dr. Arun Jayakar.S¹, Nandhinisri.M², Niranjan.T³, Swetha.P⁴

¹Assistant Professor, Department of Electronics and Instrumentation Engineering, Bannari Amman Institute of Technology, Sathyamangalam, India.

^{2,3,4}Final Year Student, Department of Electronics and Instrumentation Engineering, Bannari Amman Institute of Technology, Sathyamangalam, India.

Abstract -Many industries like the petroleum industry, food manufacturing, and others rely on the control of liquid levels for their stability. Many industries, especially those that deal with chemicals, utilize conical tanks because they prevent the accumulation of solid in the bottom of the tanks. There is a need for the control of liquid level in manufacturing processes. Controlling nonlinear processes is quite difficult. Conical tanks work well for both solid and liquid mixtures because of their unique, unsymmetrical shape. A key element in the design of the inverted conical tank is that it does not have a linear design and that it has no fixed cross section. This paper proposes the design of a conical tank, System identification, mathematical model and to design Internal Model Control (IMC) and Model Predictive Controller (MPC) controllers and to check the robustness by using MATLAB Simulink, where the controller will be simulated. .

Key Words: Conical Tank, Mathematical modeling, System Identification, IMC, MPC, Robustness.

1. INTRODUCTION

The process industries face basic issues of controlling the liquid level and flow in between tanks. A conical tank is one of the most important applications in process industries. Nonlinear dynamics, parameterization, and other simulation aspects will be presented. Our methodology is to employ input and output models. Industries prefer discrete time units over all other units of time because of the extensive use of digital computers. To apply the models as far as possible, a mathematical model is needed. If the processes and equations are not understood, or cannot be described precisely, then the structure and parameters must be identified by experimentation.

This project covers the following subjects: 1. Models of nonlinear dynamic processes 2. Recognition test signals 3. Techniques for parameter estimation 4. Strategies for determining nonlinearity, 5. Identification of the structure 6. Validation of the model, 7. Case studies for identifying real-world processes.

Models can be used to explain the behaviour of complex systems. These models are used for a variety of purposes, including prediction, description, process optimization, operator training, fault detection, and controller design.

The dynamic control architecture most often incorporates an input/output model. Dynamic models of dynamic systems are being developed using first-principles from physics, chemistry, biology, and economics, among other disciplines. Due to the specialized process expertise needed, these "physical" models are often difficult to obtain. Additionally, the sophistication of the majority of real-world systems, combined with the lack of information about several parameters, results in inaccurate models.

An alternative method for obtaining a dynamic model is known as system identification, which is concerned with the development of mathematical models for dynamic systems using calculated I/O data. Model construction through system identification entails the collection and processing of I/O-data in order to determine an acceptable model structure and provide a parametric or non-parametric model definition.

2. SYSTEM DESCRIPTION

2.1. CONICAL TANK SYSTEM:

The conical tank system's cross section varies significantly from top to bottom, making it a highly nonlinear operation as shown in Fig.2.1. The system factor is considered and the parameters are adjusted accordingly. Adjusting and fixing the outlet flow maintains the optimal output volume. The tank system that will be stimulated has a single input and output. The desired level is denoted by 'h' and is maintained by operating the system's inlet flow 'Q_{in}'. In this process, the variables 'h' and 'Q_{in}' are referred to as the control and manipulated variables, respectively.



Fig 2.1. Conical tank system

2.2. BLOCK DIAGRAM:

The Block diagram of conical tank system is shown in Fig 2.2. Using various models such as transfer function model, state space model and process model the system is validated tested for the output.

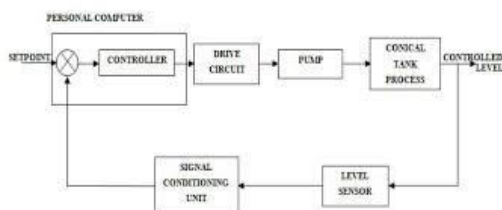


Fig 2.2. Block diagram of conical tank system

2.3. PROCESS MODEL OF A CONICAL TANK SYSTEM:

The process model of the proposed system as shown in Fig 2.2.1 includes level transmitter, current to voltage converter, voltage to current converter, current to pressure converter, data acquisition cord, Differential Pressure Transmitter(DPT), pneumatic valve.

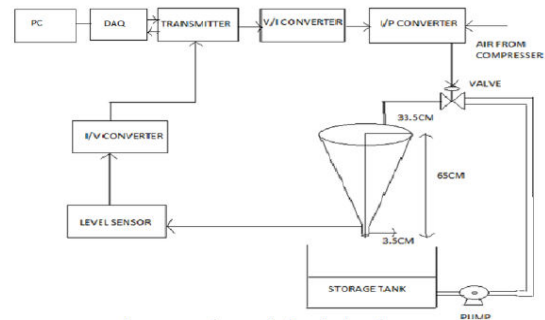


Fig 2.2.1.The process model of the proposed system.

2.4. MATHEMATICAL MODELLING:

Specification of conical tank system:

Conical Tank is of height 65 cm and its top diameter is 33.5, Bottom diameter is 3.5cm made of Stainless Steel. **Pump** with Centrifugal 0.5HP and **Control Valve** with size ¼ Pneumatic actuated with Air to open type; and Input: 3-15PSI. Range of Rota meter is 0-460 LPH.

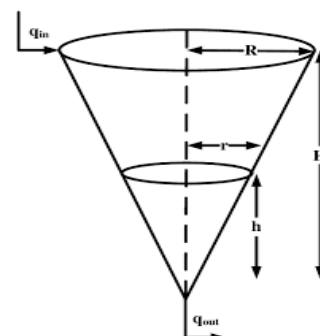


Fig.2.4.Conical tank system

Derivation of the conical tank

q_{in} – In flow,

q_{out} – Out flow,

b – Valve coefficient,

H – Height of the tank,

R – Radius of the tank,

h – Height of the water in the tank,

r – Radius of the water in the tank.

$$q_{in} - q_{out} = \frac{A(h_1)dh_1}{dt} \dots\dots (1)$$

$$\frac{dh_1}{dt} = \frac{q_{in} - b\sqrt{h_1}}{\pi R^2 h_1^2 / H^2} \dots\dots\dots (2)$$

The mathematical model is obtained by integrating equation (2),

$$h_1 = \int (q_{in} - b\sqrt{h_1}) * \frac{1}{\pi} * \frac{H^2}{h^2} * \frac{1}{R^2} \dots\dots\dots (3)$$

The transfer function is obtained for controlling of conical tank system.

$$G(s) = \frac{64.08e^{-0.4s}}{960s+1} \dots\dots\dots (4)$$

Using the above transfer function the system is validated using various models for best fit through MATLAB software.

3. SYSTEM IDENTIFICATION:

Building of mathematical models for a dynamic process by measuring system's input and output signals. The objective of system identification is to build a stochastic or non-parametric model of a real system solely from measured I/O data that accurately reproduces the static and dynamic I/O behaviour of the latter when subjected to external influences, even in the presence of noise corrupted data. The identification of a system includes the following steps:

1) Data acquisition, 2) model structure selection and determination, 3) parameter estimation, and 4) model validation.

Using an acceptable inputs in the experiment would be critical for good data acquisition. Most measurement tools are digital today. Since it takes 6 to 10 times the device bandwidth to sample the I/O data, the sample rate of the I/O data has to be chosen at 6 to 10 Hz.

In this Paper, The identified system is validated using different models using MATLAB and its responses with respect to frequency domain, its residual analysis, Transient responses and model output graphs such as process model, state space model and transfer function model are taken and analysed for best fit from which the best model is identified.

The mathematical modelling is done using open loop test and the transfer function is obtained from which the validation of the system identification process is done using MATLAB.

4. IMC and MPC Controller:

The robustness and the performance of the controller for a common purpose use IMC tuning rules which gives an acceptable yield. However the disadvantage of IMC tuning rules involves disturbances when the desired closed loop dynamics is faster than the process dynamics. This problem was addressed by Morari et al at the year of 1996 which includes integrated for the performance during IMC designing at the output disturbance side. The problem faced by many processes such as low disturbance suppression was dealt by Morari which was also implied to MPC model.

The fundamental open loop IMC structure is illustrated in Fig.4.1, where model uncertainty and disturbance are ignored, and $q(s)$ and $g_p(s)$ denote the controller and plant transfer functions, respectively. The relationship between the set-point $r(s)$ and the output $y(s)$ is as follows:

$$y(s) = g_p(s)q(s)r(s).$$

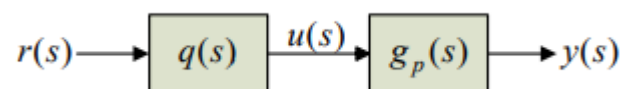


Fig.4.1. The open-loop IMC Structure.

Ensuring robustness while improving dynamic stability and decreasing static errors When they introduced the process version (the copy of the process), they named it “internal model” $g_p(s)$ as shown in Fig.4.2. This in-phase model is linked to the original systems, to produce modified error signal for the controller.

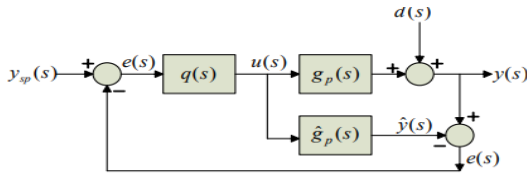


Fig.4.2. The closed-loop IMC Model.

From the closed loop diagram the transfer function for the IMC based controller is obtained as follows:

$$G(s) = \frac{64.08e^{-0.4s}}{960s+1} \dots\dots (1)$$

$$G(s)_{IMC} = \frac{1}{G(s)} S(s) \dots\dots (2)$$

By solving the IMC transfer function is obtained:

$$G(s)_{IMC} = \frac{960s+1}{64.08s} \dots\dots (3)$$

Model Predictive Control (MPC) is a more refined control method that satisfies a variety of conditions. Since the 1980s, it has been widely accepted in the process industries. MPC has the capacity to anticipate future events and takes advantage of it to take corrective action before it happens.

The MPC Procedure is explained in Fig.4.3. where N_p is the prediction horizon, $u(t+k/t)$ is the predicted control action at $t+k$ given $u(t)$. Similarly, $y(t+k/t)$ is the predicted output at $t+k$ given $y(t)$.

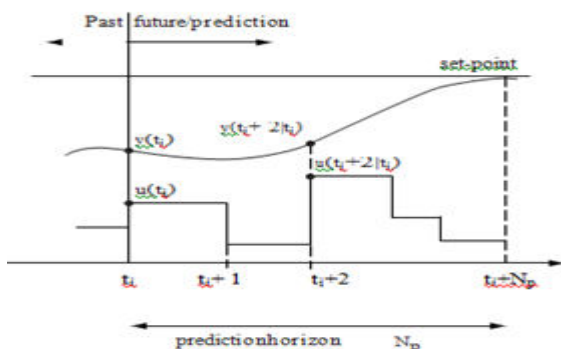


Fig.4.3. The MPC Procedure

$i=1$

$$y^{\wedge}(k+J) = S_j \Delta u(k) + y^{\wedge 0}(k+J) \dots\dots\dots (2)$$

subs $y^{\wedge}(k+J) = y_{sp}$

The predictive controller can be rearranged to produce the desired results.

$$\Delta u(k) = y_{sp} - y^{\wedge 0}(k+J) / s_j \dots\dots\dots (3)$$

The law in 3rd equation can be predicts on the expectation of things in the future, and the inverse of the predictive model would be interpreted by control law.

5. RESULTS

5.1. VALIDATION OF IDENTIFIED SYSTEM:

For validating the best fit for the non-linear Conical tank system, different graphical analysis are taken for a better outcome. Here 3 signals namely chirp signal, step signal and signal generator is taken for comparing their best fit for the Conical tank system.

The following are the block diagram of chirp signal, step signal and signal generator.

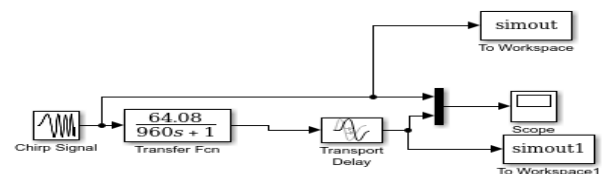


Fig.5.1.1. Block diagram of Chirp signal

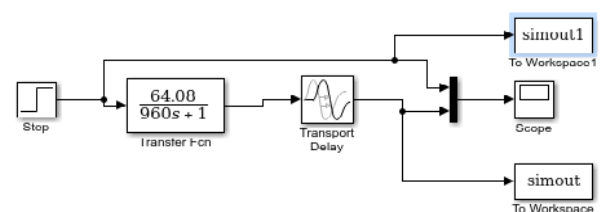


Fig.5.1.2. Block diagram of Step signal

N-1

$$Y^{\wedge}(k+1) = \sum S_i \Delta u(k-i+1) + S_N u(k-N+1) \dots\dots\dots (1)$$

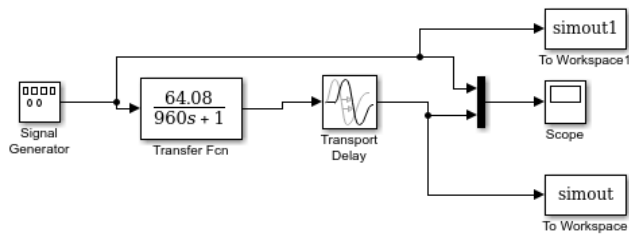


Fig.5.1.3.Block diagram of Signal generator

The frequency response of the 3 signals: chirp signal, step signal and signal generator are shown in below Figs.

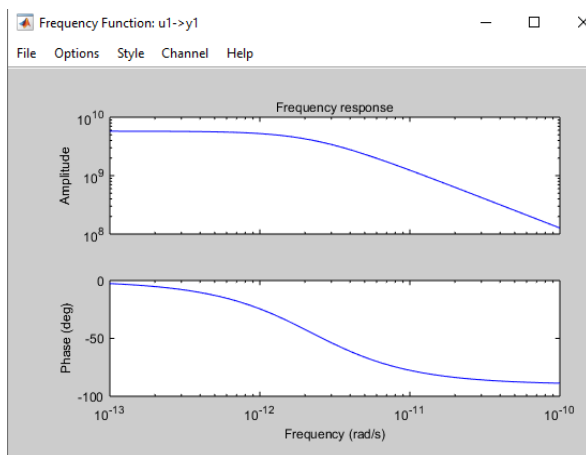


Fig.5.1.4.Frequency response of chirp signal

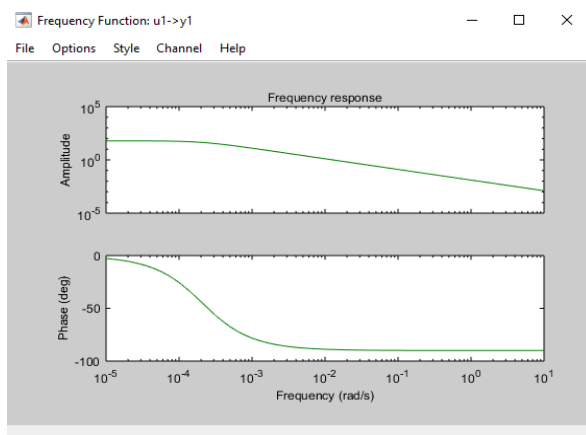


Fig.5.1.5.Frequency response of Step Signal

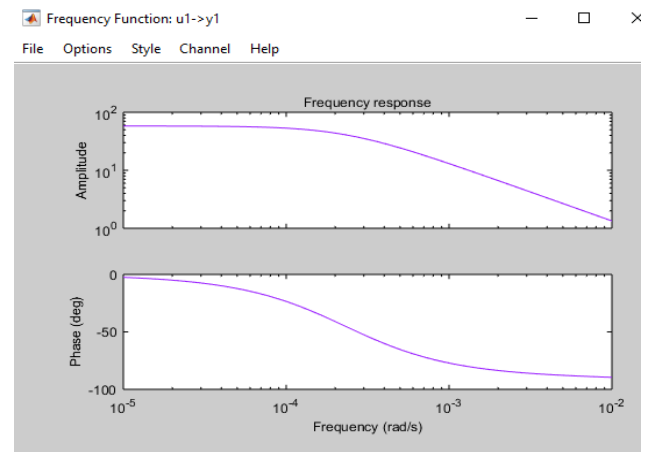


Fig.5.1.6.Frequency response of Signal generator.

The model output of the 3 signals, chirp signal, step signal and signal generator are shown in below Figs. This shows the best fit of the measured and simulated model output.

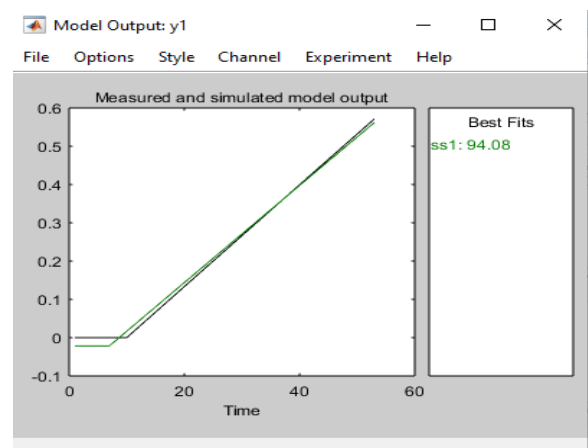


Fig.5.1.7.Model output of chirp signal

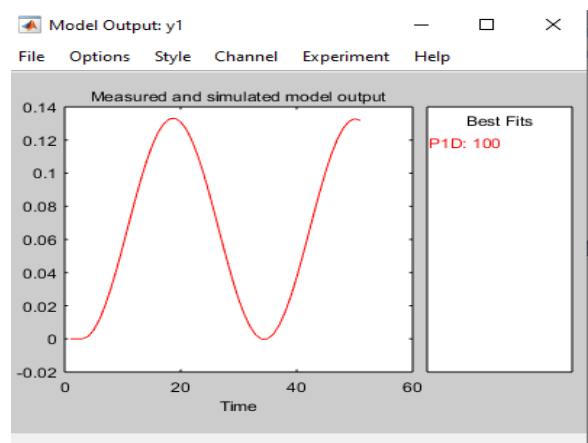


Fig.5.1.8.Model output of step signal

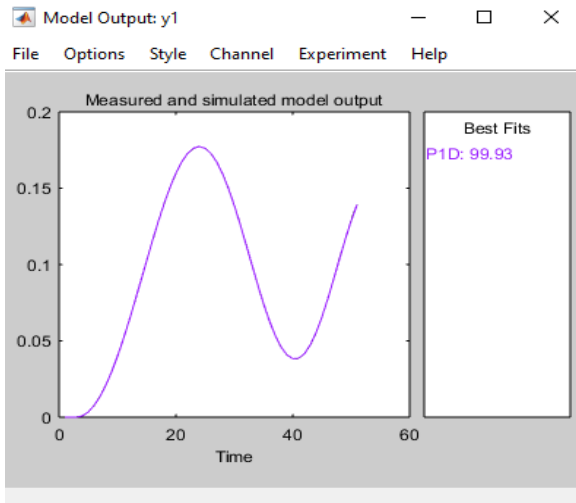


Fig.5.1.9. Model output of signal generator

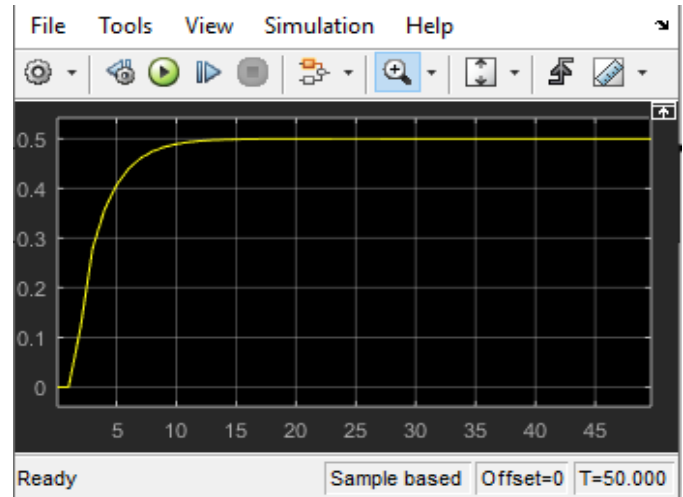


Fig.5.2.2. Scope output of IMC model.

The best fits of the chirp signal, step signal and signal generators are 94.08, 100 and 99.93 respectively. Comparing the model outputs of the signals the best fit to the conical tank system is step signal which shows 100% fit to the non-linear system.

5.2. IMC MODEL

The IMC model is designed and performance analysis for the model is done to know the better settling time. The block diagram of IMC model in MATLAB is shown in the Fig.5.2.1.

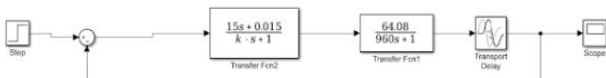


Fig.5.2.1. Block diagram of IMC model design

From the graphical analysis as shown in Fig.5.1.2, it is clear that IMC model has a better settling time and better performances.

The MPC model is designed and performance analysis for the model is done to know the better settling time. The block diagram of MPC model in MATLAB is shown in the Fig.5.2.3.

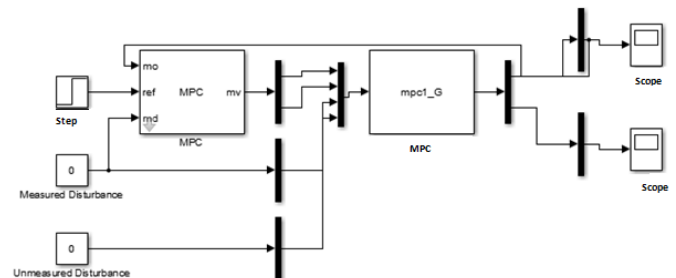


Fig.5.2.3. Block diagram of MPC model design

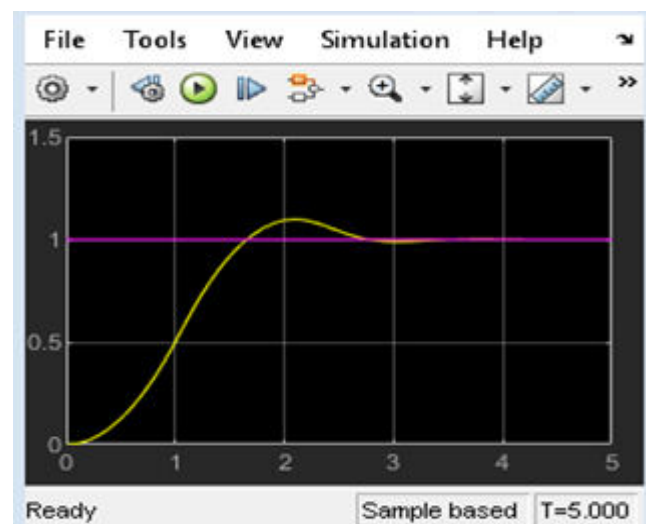


Fig.5.2.4. Scope output of MPC model.

From the graphical analysis as shown in Fig.5.2.4, it is clear that MPC model has a better settling time and better performances.

6. CONCLUSION:

The conical tank system is considered for the non-linear system and using the mathematical modelling the transfer function is obtained from which the system is validated for its fitness. For validating three signals are considered and transfer function model is taken for obtaining best fitness value and in addition graph output such as frequency output and the model output is taken. All the 3 model's output (best fit) are in the range of 90-100, which depicts that the non-linear system taken is best fit. Further IMC model is also designed and its performance is analyzed by obtaining transfer function and the scope output clearly shows IMC model has better settling time and better performances.

7. REFERENCE:

1. Janaki.M, Soniya.V, Arunkumar.E, "Design and Comparative analysis of controller for non-linear tank system", International Research Journal of Engineering and Technology, Vol 2, Issue 3, 2016.
2. K. Vivetha, K. Gandhimathi, T. Praveena, D. Angeline, "Model based controller design for a conical tank system", International Journal of Computer Applications, Vol 85-No 12, 2014.
3. P. Naveen Kumar, T. Anitha, "Design of Optimal controller for conical tank system", International Journal of Innovative research in Electrical, Electronics, Instrumentation and control Engineering, vol 4, issue 2, 2016.
4. B.W. Bequette, "Chapter-7 IMC-based Tuning", 1999.
5. S.R. Warier, Venkatesh Sivanandam, "Design of controllers based on MPC for a conical tank system", Advances in Engineering, Science and Management, 2012.
6. Nithya Venkatesan, N. Anantharaman, "Controller based design based on model predictive control for a non linear process" IEEE, 2012.
7. S. Vadivazhagi, Dr. N. Jaya, "Modelling and Stimulation of Conical tank System", International Journal of Innovative research in Science, Engineering and Technology. Vol.3,

Issue 6, 2014.

8. Aravind Pitchai Venkataraman, M. Valluvan, M. Saranya, "Simulation based Modelling and Implementation of Adaptive control Technique for non linear tank", International Journal of Computer Applications, 2013.